

NEUTRINOLESS DOUBLE BETA DECAY IN HEAVY DEFORMED NUCLEI*

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Abstract

The zero neutrino mode of the double beta decay in heavy deformed nuclei is investigated in the framework of the pseudo SU(3) model, which has provided an accurate description of collective nuclear structure and predicted half-lives for the two neutrino mode in good agreement with experiments. In the case of ^{238}U the calculated zero neutrino half-life is at least three orders of magnitude greater than the two neutrino one, giving strong support of the identification of the radiochemically determined half-life as being the two neutrino double beta decay. For ^{150}Nd the zero neutrino matrix elements are of the order of magnitude of, but lesser than, those evaluated using the QRPA. This result confirms that different nuclear models produce similar zero neutrino matrix elements, contrary to the two neutrino case. Using these pseudo SU(3) results and the upper limit for the neutrino mass we estimate the $\beta\beta_{0\nu}$ half-lives for six nuclei. An upper limit for majoron coupling constant is extracted from the experimental data.

1 Introduction

Many grand unified theories [1, 2, 3] regard neutrinos as Majorana particles, identical to its own antiparticle, predicting light neutrino masses and the existence of right-handed currents. In order to get information about this physics beyond the standard model many extensive experimental investigations have been performed to detect the neutrinoless mode of the double beta decay ($\beta\beta_{0\nu}$).

Double beta decay can be classified into various modes according to the light particles besides the electrons associated with the decay. The two neutrino mode ($\beta\beta_{2\nu}$), in which two electrons

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and two neutrinos are emitted, takes place independently of the neutrino properties, and it is the only one which has been measured up to now [4, 5, 6, 7, 8, 9, 10].

The nonconserving lepton number zero neutrino mode ($\beta\beta_{0\nu}$), in which only two electrons are emitted, can occur if, and only if, the neutrino is a massive Majorana particle [11], *i.e.* if ν and $\bar{\nu}$ are just the left and right handed helicity states of a single particle state ν^M . For this reason this mode, unobserved up to now, concentrates the interest concerned with new physics, beyond the standard model.

Another proposed double beta decay mode is the decay in two electrons and a massless neutral pseudoscalar χ , known as the majoron. If this $\beta\beta_{0\nu\chi}$ decay competes with the $\beta\beta_{2\nu}$ mode, the electron energy spectra of the two emitted electrons must be compared in detail in order to distinguish both decay modes.

In previous papers [12] we used the pseudo SU(3) shell model to evaluate the two neutrino double beta half lifes of eleven heavy deformed potential double beta emitters. We found good agreement with the available experimental information. The radiochemically measured ^{238}U decay [10] raised some expectatives, given the experimental array cannot discriminate between the different double beta decay modes, and some theoretical estimations[13] predicted similar decay ratios for both the 0ν and the 2ν modes. Our calculations for the two neutrino mode were consistent with the experimental result. In the present work we estimate the zero neutrino matrix elements for six heavy deformed double beta emitters, including ^{238}U and ^{150}Nd . In the case of ^{238}U we found the zero neutrino half-life at least three orders of magnitude greater than the two neutrino one, giving strong support of the identification of the observed half-life as being the two neutrino double beta decay. Experiments in ^{150}Nd have set the lowest limits to the majoron coupling constant. For ^{150}Nd the $\beta\beta_{0\nu}$ matrix elements are of the order of magnitude of, but lesser than, those evaluated using the QRPA. This result confirms the experience that different nuclear models produce similar zero neutrino matrix elements, contrary to the two neutrino case [2, 3, 13, 14, 15]. Using these pseudo SU(3) results and the upper limit for the neutrino mass we estimate the $\beta\beta_{0\nu}$ half-lives for six nuclei. An upper limit for the majoron coupling constant is also extracted from the experimental data.

In section 2 we review the double beta decay formalism for the zero neutrino and majoron emitting modes. A summary of the pseudo SU(3) model is given in Section 3 and in Section 4 the calculations of the nuclear matrix elements $M_{0\nu}$ within this model is presented. Section 5 contains the results and discussions and Section 6 the conclusions. In Appendix A some details of the evaluation of the radial integrals are given, the connection with the 2ν formalism is presented in Appendix B. The inclusion of short range correlations and the effects related with the finite nucleon size are discussed in Appendix C.

2 Double beta decay

In the case the 0ν decay exists, the virtual neutrino must be emitted in one vertex, and absorbed in the other. Since in the standard theory the emitted particle is a right-handed antineutrino and the absorbed one a left-handed neutrino the process requires that a) the exchanged neutrino is a Majorana particle and b) both neutrinos have a common helicity component. The helicity matching can be satisfied in two ways: a) the neutrinos have a nonvanishing mass and therefore a “wrong” helicity component proportional to m_ν/E_ν . The decay rate will be proportional to $\langle m_\nu \rangle^2$. Or b) the helicity restriction could be satisfied if there is a right handed current interaction. In this case a nonvanishing mass allowing mixing of neutrino types is also required

[11, 16].

For massive Majorana neutrinos one can perform the integration over the four-momentum of the exchanged particle and obtain a “neutrino potential” which for a light neutrino ($m_\nu < 10\text{MeV}$) has the form

$$H(r, \bar{E}) = \frac{2R}{\pi r} \int_0^\infty dq \frac{\sin(qr)}{q + \bar{E}} \quad (1)$$

where \bar{E} is the average excitation energy of the intermediate odd-odd nucleus and the nuclear radius R has been added to make the neutrino potential dimensionless. In the zero neutrino case this closure approximation is well justified [17]. The final formula, restricted to the term proportional to the neutrino mass, is [3, 11]

$$(\tau_{0\nu}^{1/2})^{-1} = \left(\frac{< m_\nu >}{m_e} \right)^2 G_{0\nu} M_{0\nu}^2. \quad (2)$$

where $G_{0\nu}$ is the phase space integral associated with the emission of the two electrons and the nuclear matrix elements $M_{0\nu}$ are discussed below.

The double beta decay with majoron emission has been investigated in various models[3, 11]. The original Gelmini-Roncadelli one[18] is incompatible with the LEP measurement of the invisible width of the Z boson. However, various suitable modifications have been considered[16, 19]. The decay rate for the majoron emitting mode of the double beta decay resembles the 0ν mode:

$$(\tau_{0\nu,\chi}^{1/2})^{-1} = < g_{\nu,\chi} >^2 G_{0\nu,\chi} M_{0\nu}^2. \quad (3)$$

where $< g_{\nu,\chi} >$ is the effective majoron neutrino coupling constant, $G_{0\nu,\chi}$ is the phase space integral which describes two electrons plus the massless majoron, and the nuclear matrix elements are *the same* in both cases.

These nuclear matrix elements are [11]

$$M_{0\nu} \equiv |M_{0\nu}^{GT} - \frac{g_V^2}{g_A^2} M_{0\nu}^F| \quad (4)$$

with

$$M_{0\nu}^\alpha = < 0_f^+ || O^\alpha || 0_i^+ > \quad (5)$$

where the kets $|0_i^+>$ and $|0_f^+>$ denote the corresponding initial and final nuclear states, the quantities g_V and g_A are the dimensionless coupling constants of the vector and axial vector nuclear currents, and

$$\begin{aligned} O^{GT} &\equiv \sum_{m,n} O_{mn}^{GT} = \sum_{m,n} \vec{\sigma}_m t_m^- \cdot \vec{\sigma}_n t_n^- H(|\vec{r}_m - \vec{r}_n|, \bar{E}) \\ O^F &\equiv \sum_{m,n} O_{mn}^F = \sum_{m,n} t_m^- t_n^- H(|\vec{r}_m - \vec{r}_n|, \bar{E}). \end{aligned} \quad (6)$$

being $\vec{\sigma}$ the Pauli matrices related with the spin operator and t^- the isospin lowering operator, which satisfies $t^-|n> = |p>$. The superindex GT denotes the Gamow-Teller spin-isospin transfer channel, while the F indicates the Fermi isospin one.

It must be noted that only the difference between Fermi and Gamow-Teller matrix elements appears in the combination $M_{0\nu}$, which can be evaluated from the new operator

$$O = \sum_{m,n} t_m^- t_n^- H(|\vec{r}_m - \vec{r}_n|, \overline{E}) (\vec{\sigma}_m \cdot \vec{\sigma}_n - (\frac{g_V}{g_A})^2) \quad (7)$$

Although $(\frac{g_A}{g_V})^2 = 1.5$ for free nucleons [13, 14], it is common to assume $(\frac{g_A}{g_V})^2 = 1.0$ in order to improve the agreement with the observed Gamow-Teller transition strength, which is quenched by roughly a factor one half in comparison with the theoretical sum rule [20]. In this way, channels which are out of the most usual models, as the delta-isobar-hole excitations, are included in an effective form [21]. Under this assumption

$$\begin{aligned} O &= \sum_{m,n} t_m^- t_n^- H(|\vec{r}_m - \vec{r}_n|, \overline{E}) (\vec{\sigma}_m \cdot \vec{\sigma}_n - 1) \\ &= -4 \sum_{m,n} t_m^- t_n^- H(|\vec{r}_m - \vec{r}_n|, \overline{E}) \hat{P}_{mn}(S=0) \end{aligned} \quad (8)$$

where $\hat{P}_{mn}(S=0)$ is the projector of a two-particle state coupled to spin $S=0$. This result will be useful in order to simplify the expressions for the nuclear matrix element $M_{0\nu}$.

The finite nucleon size and the two-nucleon short range correlations in the nucleon-nucleon interaction are introduced in these matrix elements. Their explicit expression are given in Appendix C.

As mentioned above, we will use the pseudo $SU(3)$ scheme to describe the ground state of the six heavy deformed nuclei considered in this contribution. A detailed description of this formalism can be found elsewhere [12, 22, 23]. In the next section we give a brief summary of the model.

3 The Pseudo $SU(3)$ Model

A tractable shell model theory for deformed nuclei requires a severe truncation of the spherical model basis. A successful truncation can be achieved if the basis selection is made relative to those parts of the interaction that dominate the low-energy structure: the pairing and quadrupole-quadrupole ones. In the case of rotational nuclei, it has been shown that the long range part, *i.e.* the quadrupole-quadrupole interaction, is the most important one [23].

For heavy nuclei, the major shells are built by the orbitals $j = \{1/2, 3/2, \dots, \eta - 1/2\}$, which are called of normal parity plus a single particle state $j = \eta + 3/2$ of a different parity. Then we are restricting the Hilbert space to the valence neutron and proton shells constituted by the corresponding normal parity orbitals and the abnormal or unique parity levels.

In the pseudo $SU(3)$ shell model coupling scheme [22] the normal parity orbitals are identified with orbitals of a harmonic oscillator of one quanta less $\tilde{\eta} = \eta - 1$. This set of orbitals, with $\tilde{j} = j = \tilde{l} + \tilde{s}$, pseudo spin $\tilde{s} = 1/2$, and pseudo angular momentum taking the values $\tilde{l} = \tilde{\eta}, \tilde{\eta} - 2, \dots, 1$ or 0 define the so called pseudo space and only recently it was found an analytic expression for the transformation that take us from the normal parity orbitals to the pseudo space [24]. Applying this transformation to the spherical Nilsson hamiltonian it can be shown explicitly that the strength of the pseudo spin orbit interaction is almost zero and the orbitals $j = \tilde{l} \pm 1/2$ are nearly degenerate doublets. For the configurations of identical particles occupying a single j orbital of abnormal parity, a convenient characterization of states is made by means of the seniority coupling scheme.

The many particle states of n_α nucleons in a given shell η_α , $\alpha = \nu$ or π , can be defined by the totally antisymmetric irreducible representations $\{1^{n_\alpha^N}\}$ and $\{1^{n_\alpha^A}\}$ of unitary groups

of dimensions corresponding to the sizes of the normal $\Omega_\alpha^N = (\tilde{\eta}_\alpha + 1)(\tilde{\eta}_\alpha + 2)$ and unique $\Omega_\alpha^A = 2\eta_\alpha + 4$ parity spaces, respectively; with the constraint $n_\alpha = n_\alpha^A + n_\alpha^N$.

A complete classification of the states can be defined by the following chains of groups:

$$\begin{aligned} \{1^{n_\alpha^N}\} \quad \{\tilde{f}_\alpha\} \quad \{f_\alpha\}\gamma_\alpha(\lambda_\alpha, \mu_\alpha) \quad \tilde{S}_\alpha \quad \tilde{K}_\alpha \quad \tilde{L}_\alpha \quad J_\alpha^N \\ U(\Omega_\alpha^N) \supset U(\Omega_\alpha^N/2) \times U(2) \supset SU(3) \times SU(2) \supset SO(3) \times SU(2) \supset SU_J(2) \end{aligned} \quad (9)$$

$$\begin{aligned} \{1^{n_\alpha^A}\} \quad v_\alpha \quad \beta_\alpha \quad J_\alpha^A \\ U(\Omega_\alpha^A) \supset Sp(\Omega_\alpha^A) \supset SU(2) \end{aligned} ,$$

where above each group the quantum numbers that characterize its irreducible representations (irreps) are given and the γ_α , β_α and \tilde{K}_α are multiplicity labels of the indicated reductions. For the normal parity spaces the pseudo LS coupling scheme is used and the following relations between its quantum numbers are satisfied

$$n_\alpha^N = f_\alpha^1 + f_\alpha^2, \quad S_\alpha = 1/2(f_\alpha^1 - f_\alpha^2), \quad \{\tilde{f}_\alpha\} = \{2^{n_\alpha^N/2 - S_\alpha}, 1^{2S_\alpha}\} , \quad (10)$$

while for the abnormal parity spaces the seniority configurations v_α are appropriate.

Proton and neutron states are coupled in angular momentum in both the normal and unique parity sectors, generating states with angular momentum J^N and J^A respectively. The wave function of the many-particle state with angular momentum J and projection M is expressed as a direct product of the normal and unique parity ones, as:

$$|JM\rangle = \sum_{J^N J^A} [|J^N\rangle \otimes |J^A\rangle]_M^J \quad (11)$$

We will be interested in the ground states of some deformed nuclei, which have $J = M = 0$.

For even-even heavy nuclei, it has been shown that if the residual neutron-proton interaction is of the quadrupole type, independently of the interaction in the proton and neutron spaces, for yrast states below the back bending region the most important normal parity configurations are those with highest spatial symmetry $\{\tilde{f}_\alpha\} = \{2^{n_\alpha^N/2}\}$. This implies that $\tilde{S}_\pi = \tilde{S}_\nu = 0$, that is, only pseudo spin zero configurations are taken into account.

Additionally in the abnormal parity space only seniority zero configurations, $v_\pi = v_\nu = 0$, are taken into account. This simplification implies that $\beta_\pi = \beta_\nu = 1$ and $J_\pi^A = J_\nu^A = 0$. This is a very strong assumption, which in future works is expected to be improved, but is quite useful in order to simplify the calculations.

As an example, the ground-state band of ^{238}U is characterized by the eigenstates

$$\begin{aligned} | \{1^6\}_\pi \{2^3\}_\pi (18, 0)_\pi; \{1^{12}\}_\nu \{2^6\}_\nu (36, 0)_\nu; 1(54, 0) KJM >_N \\ | (i_{13/2})_\pi^4, J_\pi^A = M_\pi^A = 0; (j_{15/2})_\nu^8, J_\nu^A = M_\nu^A = 0 >_A \end{aligned} , \quad (12)$$

where the first term indicates the normal part and the second the abnormal one. The ^{238}U ground state has $K = 1, J = M = 0$. For a detailed description of the way this state was constructed the reader is referred to [12, 23].

4 Calculation of the matrix elements $M_{0\nu}$

We want to evaluate the nuclear matrix element (4) for several heavy deformed nuclei, which are potential $\beta\beta$ emitters. According to the above discussion the initial $|0_i^+\rangle$ and final $|0_f^+\rangle$ ground states are of the type indicated in the Eq.(12).

We expand the two body transition operator in order to obtain its normal and abnormal parity components

$$\begin{aligned} O^\alpha &= -\sum_{j_1, j_2} \sum_{j_3, j_4} \sum_{J, M} \langle (j_1 j_2) JM | O_{12}^\alpha | (j_3 j_4) JM \rangle [a_{j_1}^\dagger a_{j_2}^\dagger]^{JM} [a_{j_3} a_{j_4}]^{JM} \\ &= -\sum_{j_\pi, j_{\pi'}} \sum_{j_\nu, j_{\nu'}} \sum_{J, M} \langle (j_\pi j_{\pi'}) JM | O_{12}^\alpha | (j_\nu j_{\nu'}) JM \rangle [a_{j_\pi}^\dagger a_{j_{\pi'}}^\dagger]^{JM} [a_{j_\nu} a_{j_{\nu'}}]^{JM} \end{aligned} \quad (13)$$

where in the second line only the nonvanishing contributions were retained. Transforming this operator to the pseudo $SU(3)$ space, we have formally the expression

$$O^\alpha = O_{N_\pi N_\nu}^\alpha + O_{N_\pi A_\nu}^\alpha + O_{A_\pi N_\nu}^\alpha + O_{A_\pi A_\nu}^\alpha \quad (14)$$

where the subindices NN, NA, \dots are indicating the normal or abnormal spaces of the fermion creation and annihilation operators, respectively. By using the Nilsson scheme in order to obtain the occupation number we are grouping nucleons in pairs. For this reason only these four cases can give contribution different from zero.

In the present work we will restrict our analysis to six potential double beta emitters which, within the approximations of the pseudo $SU(3)$ scheme, are also decaying via the 2ν mode. They include the observed $^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$ and $^{238}\text{U} \rightarrow ^{238}\text{Pu}$ decays. In this case two neutrons belonging to a normal parity orbital decay in two protons belonging to an abnormal parity one. Under the seniority zero assumption for nucleons in abnormal parity orbitals, only proton pairs coupled to $J=0$ are allowed to exist in the ground state, restricting the above summation to $J=0, M=0$. It follows that $\pi = \pi'$ and $\nu = \nu'$. The matrix elements becomes

$$\begin{aligned} M_{0\nu}^\alpha &= -\sum_{j_\pi, j_\nu} \langle (j_\pi j_\pi) J = M = 0 | O^\alpha | (j_\nu j_\nu) J = M = 0 \rangle \\ &< \{2^{n_\pi^N/2}\}(\lambda_\pi^f, \mu_\pi^f); \{2^{n_\nu^N/2-1}\}_\nu(\lambda_\nu^f, \mu_\nu^f); (\lambda^f, \mu^f) 100 | [a_{j_\nu} a_{j_\nu}]^{J=M=0} \\ &\quad | \{2^{n_\pi^N/2}\}(\lambda_\pi^i, \mu_\pi^i); \{2^{n_\nu^N/2}\}_\nu(\lambda_\nu^i, \mu_\nu^i); (\lambda^f, \mu^f) 100 \rangle \\ &< (j_\pi^A)^{n_\pi^A+2}, J_\pi^A = M_\pi^A = 0; (j_\nu^A)^{n_\nu^A}, J_\nu^A = M_\nu^A = 0 | [a_{j_\pi}^\dagger a_{j_\pi}^\dagger]^{J=M=0} \\ &\quad | (j_\pi^A)^{n_\pi^A}, J_\pi^A = M_\pi^A = 0; (j_\nu^A)^{n_\nu^A}, J_\nu^A = M_\nu^A = 0 \rangle \end{aligned} \quad (15)$$

In these cases, it is also simple to show that [12]

$$\begin{aligned} < (j_\pi^A)^{n_\pi^A+2}, J_\pi^A = M_\pi^A = 0 | [a_{j_\pi}^\dagger a_{j_\pi}^\dagger]^{J=M=0} | (j_\pi^A)^{n_\pi^A}, J_\pi^A = M_\pi^A = 0 \rangle \\ &= \delta_{j_\pi^A j_\pi} \sqrt{\frac{(n_\pi^A+2)(2j_\pi+1-n_\pi^A)}{2j_\pi+1}} \end{aligned} \quad (16)$$

The operators in the normal parity sector must be transformed to the pseudo-space. These nucleons are coupled to *pseudospin* $\tilde{S} = 0$, which together with the $J=0$ condition implies $\tilde{L} = 0$. Then [12, 23]

$$[a_{j_\nu} a_{j_\nu}]^{J=M=0} = \sqrt{\frac{2j_\nu+1}{2(2\tilde{l}_\nu+1)}} [\tilde{a}_{(0, \tilde{\eta}_\nu) \tilde{l}_\nu; \frac{1}{2}} \tilde{a}_{(0, \tilde{\eta}_\nu) \tilde{l}_\nu; \frac{1}{2}}]_{\tilde{L}=\tilde{S}=0; J=M=0} \quad (17)$$

Expanding the normal states in their proton and neutron subspaces, and the above operator in their $SU(3)$ components, together with Eq. (16), we arrive to the final expression

$$\begin{aligned}
M_{0\nu}^\alpha &\equiv \sum_{j_\nu} M_{0\nu}^\alpha(j_\nu) \\
&= - \sum_{j_\nu} \sqrt{\frac{2j_\nu+1}{2(2\tilde{l}_\nu+1)}} \sqrt{\frac{(n_\pi^A+2)(2j_\pi+1-n_\pi^A)}{2j_\pi+1}} < (j_\pi^A j_\pi^A) J = M = 0 | O^\alpha | (j_\nu j_\nu) J = M = 0 > \\
&\quad \sum_{K_\pi L_\pi} \sum_{K_\nu L_\nu} < (\lambda_\pi^f, \mu_\pi^f) K_\pi L_\pi, (\lambda_\nu^f, \mu_\nu^f) K'_\nu L_\pi \| (\lambda^f, \mu^f) 10 > \\
&\quad < (\lambda_\pi^i, \mu_\pi^i) K_\pi L_\pi, (\lambda_\nu^i, \mu_\nu^i) K_\nu L_\pi \| (\lambda^i, \mu^i) 10 > \\
&\quad \sum_{(\lambda_0, \mu_0) \rho_0} < (\lambda_\nu^i, \mu_\nu^i) K_\nu L_\nu, (\lambda_0, \mu_0) 10 \| (\lambda_\nu^f, \mu_\nu^f) K'_\nu L_\nu >_{\rho_0} \\
&\quad < (0, \tilde{\eta}_\nu) 1\tilde{l}_\nu, (0, \tilde{\eta}_\nu) 1\tilde{l}_\nu \| (\lambda_0, \mu_0) 10 > \\
&\quad < (\lambda_\nu^f, \mu_\nu^f) \| [\tilde{a}_{(0, \tilde{\eta}_\nu) \tilde{l}_\nu, \frac{1}{2}} \tilde{a}_{(0, \tilde{\eta}_\nu) \tilde{l}_\nu, \frac{1}{2}}]^{(\lambda_0, \mu_0); \tilde{S}=0} \| (\lambda_\nu^i, \mu_\nu^i) >_{\rho_0}
\end{aligned} \tag{18}$$

where the $< (\lambda_1, \mu_1) K_1 L_1, (\lambda_2, \mu_2) K_2 L_2 \| (\lambda, \mu) K L >_\rho$ are SU(3) Clebsch-Gordan coefficients, and $< (\lambda_1, \mu_1) \| O \| (\lambda_2, \mu_2) >$ are triple reduced matrix elements [12]. We have implicitly defined $M_{0\nu}^\alpha(j_\nu)$ as the contribution of each normal parity neutron state j_ν to the nuclear matrix element in the transition $(j_\nu)^2 \rightarrow (j_\pi^A)^2$.

The two-body matrix element can be expanded in its L, S components

$$\begin{aligned}
< (j_\pi j_\pi) J = M = 0 | O^\alpha | (j_\nu j_\nu) J = M = 0 > \\
= \sum_S \sqrt{(2j_\pi+1)(2j_\nu+1)} (2S+1) W(\tfrac{1}{2} j_\pi S l_\pi; l_\pi \tfrac{1}{2}) W(\tfrac{1}{2} j_\nu S l_\nu; l_\nu \tfrac{1}{2}) \\
< (l_\pi l_\pi) S \| H(r, \overline{E}) \| (l_\nu l_\nu) S > < (\tfrac{1}{2} \tfrac{1}{2}) S \| \Gamma \cdot \Gamma(\alpha) \| (\tfrac{1}{2} \tfrac{1}{2}) S >
\end{aligned} \tag{19}$$

where the $W(...; ...)$ are Racah coefficients and

$$\Gamma \cdot \Gamma(GT) \equiv \vec{\sigma}_1 \cdot \vec{\sigma}_2 \quad \Gamma \cdot \Gamma(F) \equiv 1 \quad \alpha = GT \text{ or } F. \tag{20}$$

In order to evaluate the spatial matrix elements, we use the Bessel-Fourier expansion of the potential [25], which gives

$$\begin{aligned}
< (l_1 l_2) L M | V(r) | (l_3 l_4) L M > \\
= \sum_l (-1)^{l_1+l_4+L} (l_1 \| C_l \| l_3) (l_2 \| C_l \| l_4) W(l_1 l_2 l_3 l_4; L l) (2l+1) R^l(l_1 l_2, l_3 l_4)
\end{aligned} \tag{21}$$

with $(l_1 \| C_l \| l_3)$ the reduced matrix elements of the unnormalized spherical harmonics $C_{lm}(\Omega) \equiv \sqrt{\frac{4\pi}{2l+1}} Y_{lm}(\Omega)$ and $R^l(l_1 l_2, l_3 l_4)$ the radial integrals which are explained in Appendix A.

Although we will present results for $M_{0\nu}^F$ and $M_{0\nu}^{GT}$, we have shown that using $\frac{g_A^2}{g_V^2} = 1.0$ only the $S = 0$ channel survive, *i.e.* only states with $L=0$ are of interest and, after some manipulations, we obtain the simplified expression

$$\begin{aligned}
< (j_\pi j_\pi) J = M = 0 | O | (j_\nu j_\nu) J = M = 0 > \\
= -2 \sqrt{\frac{(2j_\pi+1)(2j_\nu+1)}{(2l_\pi+1)(2l_\nu+1)}} < (l_\pi l_\pi) L = 0 | H(r, \overline{E}) | (l_\nu l_\nu) L = 0 > \\
= -2 \sqrt{(2j_\pi+1)(2j_\nu+1)} \sum_l (-1)^l (l_\nu 0 l_\pi 0 | l 0)^2 R^l(l_\pi l_\pi, l_\nu l_\nu).
\end{aligned} \tag{22}$$

The radial integrals $R^l(l_\pi l_\pi, l_\nu l_\nu)$ are evaluated in the way described in Appendix A, and they include finite nucleon size and short range correlations, as explained in Appendix C.

5 Results and discussion

We evaluated the matrix elements of $M_{0\nu}$ between the ground states of six different initial and final nuclei for the $\beta\beta$ transitions. There exists a relevant dependence of this transition amplitudes with the partition $\{n_\pi^A, n_\pi^N, n_\nu^A, n_\nu^N\}$ of the valence nucleons of each pair of nuclei. We used the standard procedure to fix them, taking deformation from experiment, and filling each Nilsson level with a pair of particles in order of increasing energies [12, 23]. As mentioned in the Introduction, we restricted the present analysis to nuclei in which the unique and normal proton and neutron valence occupations are related by

$$\begin{aligned} n_{\pi,f}^A &= n_{\pi,i}^A + 2, & n_{\nu,f}^A &= n_{\nu,i}^A \\ n_{\pi,f}^N &= n_{\pi,i}^N, & n_{\nu,f}^N &= n_{\nu,i}^N - 2 \end{aligned} \quad (23)$$

The six nuclei fulfilling these conditions are listed in Table 1, and are the same for which finite half-lives were predicted in the two neutrino case [12]. Their matrix elements $M_{0\nu}$ are shown in the second column. The value for the integrated kinematical factor, $G_{0\nu}$ in $[\text{years}^{-1}]$ was obtained following the procedure indicated by Doi et al [26] with $g_A/g_V = 1.0$ and it is given in the third column. In the fourth column of Table 1, the energy value \overline{E} is indicated and in the last two columns the theoretical predictions (for $< m_e > = 1\text{eV}$) and experimental lower limits of the $\beta\beta_{0\nu}$ -half lives are given. It has been demonstrated that these $M_{0\nu}$ matrix elements are nearly insensitive to the exact values of the average energies if they are in the range of some MeV [17]. We choose the accepted parametrization $\overline{E} = 1.12A^{1/2}$ [27].

As mentioned above, for ^{238}U the predicted 0ν half live $\tau_{0\nu}^{1/2} = 1.0 \times 10^{24}$ years is three orders of magnitude greater than the predicted 2ν half live $\tau_{0\nu}^{1/2} = 1.4 \times 10^{21}$ years, which essentially agrees with the experimental one, confirming that the observed $\beta\beta$ decay of ^{238}U has to be the two neutrino mode. Figure 1 exhibits the $M_{0\nu}^{GT}(j_\nu)$ and $-M_{0\nu}^F(j_\nu)$ components of the Gamow-Teller and Fermi nuclear matrix elements for ^{238}U . It can be seen that all the terms contribute constructively, and that the transition is strongly dominated by a pair of neutrons in the normal parity orbital $i_{11/2}^\nu$ decaying into two protons in the unique orbital $i_{13/2}^\pi$, resembling the two neutrino case (see [12] and Appendix B).

In the case of ^{150}Nd , the pseudo SU(3) 0ν matrix element reported here is about a factor four lesser than the QRPA estimations. This is a very relevant result. First, it exhibits the stability of the neutrinoless double beta decay matrix elements evaluated in quite different nuclear models, in the case of deformed nuclei. Second, this factor of four, which is little compared with the order of magnitude variations in the 2ν theoretical estimations, is still important in order to extract the parameters $< m_e >$ and $< g_{\nu,\chi} >$. In Figure 2 the $M_{0\nu}^{GT}(j_\nu)$ and $-M_{0\nu}^F(j_\nu)$ components of the Gamow-Teller and Fermi nuclear matrix elements for ^{150}Nd are shown. As for ^{238}U all the terms contribute constructively, but in this case the transition is dominated by a pair of neutrons in the normal parity orbital $h_{9/2}^\nu$ decaying into two protons in the unique orbital $h_{11/2}^\pi$, again resembling the two neutrino case.

As can be seen in the last two columns of Table 1, the $\tau_{0\nu}^{1/2}$ predicted for $< m_\nu > = 1\text{eV}$ are at least three order of magnitude greater than the experimental limits. These results reflect the fact that at the present stage of the experimental $\beta\beta$ research, the limits $< m_\nu > \leq 1.1\text{eV}$ obtained by the Heidelberg-Moscow collaboration [28] using significative volumes of ultrapure ^{76}Ge are the most sensitives. But, if the $\beta\beta_{0\nu}$ decay is observed in ^{76}Ge , at least a second observation will be essential, and ^{150}Nd is a likely candidate to do this job [16]. In the next few years the limit for $< m_\nu >$ extracted from $\beta\beta_{0\nu}$ experiments is expected to be improved up to 0.1 eV, and ^{150}Nd is one of the selected isotopes [29].

On the other side, in extracting the majoron coupling constant the ^{150}Nd has set the lowest limit [16]. The phase integral for the two emitted electrons and the majoron, with $\frac{g_A^2}{g_V^2} = 1.0$, for this nuclei is

$$G_{0\nu\chi} = 0.466 \times 10^{-14} \text{yr s}^{-1} \quad (24)$$

and the reported lower limit for this decay is [16]

$$\tau_{0\nu\chi}^{1/2} > 5.30 \times 10^{20} \text{yr s}. \quad (25)$$

Using these numbers in Eq. (3) we obtain the limit

$$< g_{\nu,\chi} > \leq 4.1 \times 10^{-4}. \quad (26)$$

This limit is less stringent than the previous one [16] obtained with the same experimental information but employing the QRPA matrix elements $M_{0\nu}^{QRPA} = 6.075$ [15].

There are various elements which contributes to the greater cancellation of the pseudo SU(3) $M_{0\nu}$ matrix elements, as compared with the QRPA ones.

i) The uncertainty in the QRPA results itself: the other QRPA reported value for ^{150}Nd is $M_{0\nu} = 5.24$ [3], 14% lesser than the above mentioned one.

ii) The way short range correlation effects were parametrized in $H(r, \overline{E})$ in the present work is not the most common one. But both finite nucleon size (FNS) and two-nucleon short range correlations (SRC) effects together reduce the 0ν matrix elements in less than 15%.

iii) Another source of cancellation could be related with the seniority zero approximation used to describe nucleons in abnormal parity states. Removing this assumption in Eq. (18) would lead to a more general expression, not restricted to states with $J = M = 0$. Being far from trivial, this extension of the pseudo SU(3) model is under investigation. But as can be seen in Fig. 3 of [15], the $(j_\pi j_\pi)J = 0; (j_\nu j_\nu)J = 0$ channel represents by far the most relevant contribution to the 0ν matrix elements, and inclusion of other multipoles could *decrease* $M_{0\nu}$.

iv) Only one active shell was allowed for protons, and one for neutrons. This is a very strong truncation, of the same type as used in shell model calculations. Extensions of the pseudo SU(3) model which takes fully into account core excitations have shown that the SU(3) component of the wave function are the dominant ones, between 50% and 60% of the total amplitude [30]. Inclusion of these symplectic excitations could increase the present results, allowing for example transitions from neutron states to proton states *in the same shell*. Given the SU(3) dominance, these corrections are estimated to contribute at most with a factor two.

Nevertheless, at the present stage of development, the pseudo SU(3) predictions are as valid as the QRPA ones, which suffer other kind of limitations, and their differences are in some way describing the theoretical uncertainties in the extraction of the parameters $< m_\nu >$ and $< g_{\nu,\chi} >$.

6 Conclusions

Our description of the neutrinoless $\beta\beta$ decay in heavy deformed nuclei can be summarized as follows:

The pseudo SU(3) model is a very powerful machinery to describe the collective behavior of heavy deformed nuclei. Together with a model hamiltonian with spherical Nilsson single particle terms, and a residual interaction of a quadrupole-quadrupole type, plus terms including some

scalar combinations of the quadrupole and orbital angular momentum operators, it has been used to reproduce very accurately the rotational spectra of heavy deformed nuclei, including the K-band splitting, the amplitudes for transitions of the E2, M1 and M3 type and the $2\nu\beta\beta$ decay of eleven heavy deformed nuclei, with good agreement with the available experimental data.

The deformed ground states were described using the pseudo SU(3) scheme, which is able to generate maximum deformation for nucleons in normal parity orbitals, filling them into the Nilsson levels but with definite total angular momentum. These states are strongly correlated, and are dominated by the proton-neutron quadrupole-quadrupole interaction, considered the microscopic origin of deformation. The nucleons in the abnormal parity orbitals are assumed to have seniority zero, i.e. in this approximation they do not contribute actively to the quadrupole moments. This is the strongest approximation of the model, only justified in terms of its simplicity.

We have evaluated the $\beta\beta_{0\nu}$ half lives of six heavy deformed nuclei, using the pseudo SU(3) approach together with the closure approximation, obtaining an order of magnitude agreement with the QRPA estimations, and a factor four of difference. We exhibited predictions for the neutrinoless double beta half-lives assuming $< m_\nu > = 1\text{eV}$, and discussed the relevance of our results. In the case of ^{238}U this result complements those obtained for the two neutrino $\beta\beta$ decay, and it confirms the observed half-life as two neutrino in origin. Limits for the majoron coupling constant were extracted from the ^{150}Nd experimental limit, and compared with the QRPA ones.

Appendix A: The radial integrals

The radial integrals are defined as [25]

$$R^l(l_1 l_2, l_3 l_4) \equiv \int_0^\infty v(p) p^2 \left\{ \int_0^\infty r_1^2 dr_1 j_l(pr_1) R_{n_1 l_1}(r_1) R_{n_2 l_2}(r_2) \int_0^\infty r_2^2 dr_2 j_l(pr_2) R_{n_3 l_3}(r_1) R_{n_4 l_4}(r_2) \right\} dp$$

$$v(p) = \frac{2}{\pi} \int_0^\infty H(r) j_0(pr) r^2 dr \quad (27)$$

The R_{nl} are the one-particle radial harmonic oscillator wave functions and $j_0(pr)$ is the 0-th order spherical Bessel function.

Using the explicit form of $H(r, \overline{E})$, Eq. (1), we find [31]

$$v(p) = \frac{2R}{\pi} \frac{1}{p(p + \overline{E})} \quad (28)$$

The explicit expression for the radial integral is [25]

$$R^l(l_1 l_2, l_3 l_4) = (M(n_1 l_1, n_3 l_3) M(n_2 l_2, n_4 l_4))^{-1/2} \sum_{m_1, m_2, m} a_{m_1}(n_1 l_1, n_3 l_3) a_{m_1}(n_2 l_2, n_4 l_4) a_{2m}(\frac{m_1 - l}{2} l, \frac{m_2 - l}{2} l) J_m \quad (29)$$

where $m_1 = l_1 + l_3 + 2s$ and $s = 0, 1, \dots, n_1 + n_3$, and similarly to m_2 , $m = l, l + 1, \dots, \frac{m_1 + m_2}{2}$,

$$M(nl, n'l') = 2^{n+n'} n! n'! (2l + 2n + 1)!! (2l' + 2n' + 1)!!$$

$$a_{l+l'+2s}(nl, n'l') = (-1)^s \sum_{\mu, \mu' = s - \mu} \frac{n!}{\mu!(n-\mu)!} \frac{n'!}{\mu'!(n'-\mu')!} \frac{(2l+2n+1)!!}{(2l+2\mu+1)!!} \frac{(2l'+2n'+1)!!}{(2l'+2\mu'+1)!!} \quad (30)$$

and

$$J_m = (b^2/2)^m \int_0^\infty \exp\left(\frac{-p^2 b^2}{2}\right) v(p) p^{2m+2} dp \quad (31)$$

being $b = \sqrt{\frac{\hbar}{m\omega}}$ the oscillator length.

In the case $\overline{E} = 0$, we obtain

$$H(r, \overline{E}) = H(r, 0) = \frac{R}{r}$$

$$v(p) = \frac{2R}{\pi p^2} \quad J_m = \frac{\sqrt{2}R}{\pi b} \Gamma(m + 1/2) \quad (32)$$

which is a not so bad approximation to the exact result [17], obtained using a nonzero \overline{E} and including the short range correlations and finite nucleon size effects, as explained in Appendix C.

Appendix B: The two-neutrino limit

It is interesting to exhibit how the two neutrino matrix elements presented in [12] can be obtained from the present formalism. In the two neutrino case $H(r) \rightarrow 1$, and the transition operators are the double Fermi or the double Gamow-Teller ones. In this limit

$$\int_0^\infty j_l(pr_1)j_l(pr_2)v(p)p^2dp = \frac{1}{2} \int_{-1}^1 H(r)P_l(\cos\omega)d(\cos\omega) \rightarrow \delta_{l0} \quad (33)$$

where $r = |\vec{r}_1 - \vec{r}_2|$ and ω is the angle between \vec{r}_1 and \vec{r}_2 and

$$\begin{aligned} R^l(l_1l_2, l_3l_4) &\rightarrow \delta_{l0}\delta_{n_1n_3}\delta_{n_2n_4} \\ <(l_1l_2)LM|V(r)|(l_3l_4)LM> &\rightarrow \delta_{n_1n_3}\delta_{l_1l_3}\delta_{n_2n_4}\delta_{l_2l_4} \end{aligned} \quad (34)$$

where n_i are the number of nodes in the radial wave function $R_{n_i l_i}$.

Using the previous results and expressing $j_\pi = l_\pi + 1/2$, $j_\nu = l_\pi - 1/2$ we obtain

$$\begin{aligned} <(j_\pi j_\pi)J=0|O|(j_\nu j_\nu)J=0> \rightarrow \delta_{n_\pi n_\nu} \delta_{l_\pi l_\nu} 2\sqrt{l_\pi(l_\pi+2)} \\ &\sum_S (2S+1)W(\frac{1}{2}l_\pi + \frac{1}{2}Sl_\pi; l_\pi \frac{1}{2})W(\frac{1}{2}l_\pi - \frac{1}{2}Sl_\pi; l_\pi \frac{1}{2}) <(\frac{1}{2}\frac{1}{2})S\|\Gamma \cdot \Gamma\|(\frac{1}{2}\frac{1}{2})S> \\ &= \delta_{n_\pi n_\nu} \delta_{l_\pi l_\nu} \sqrt{\frac{l_\pi(l_\pi+1)}{2l_\pi+1}} \{ <(\frac{1}{2}\frac{1}{2})S=0\|\Gamma \cdot \Gamma\|(\frac{1}{2}\frac{1}{2})S=0> - <(\frac{1}{2}\frac{1}{2})S=1\|\Gamma \cdot \Gamma\|(\frac{1}{2}\frac{1}{2})S=1> \}. \end{aligned} \quad (35)$$

In the case of double Fermi transitions, *i.e.* $\Gamma \cdot \Gamma = \sum_{mn} t_m^- t_n^-$, the above described matrix elements are always zero. This result was argued heuristically in [12], and exhibit the isospin as a good quantum number for the pseudo SU(3) ground state. For double Gamow-Teller transitions, where $\Gamma \cdot \Gamma = \sum_{mn} \vec{\sigma}_m \cdot \vec{\sigma}_n t_m^- t_n^-$ we obtain

$$<(j_\pi j_\pi)J=0|O^{GT}|(j_\nu j_\nu)J=0> \rightarrow -\delta_{n_\pi n_\nu} \delta_{l_\pi l_\nu} 4\sqrt{\frac{l_\pi(l_\pi+1)}{2l_\pi+1}} \quad (36)$$

which substituted in Eq. (18) gives the $<0_f^+|\Gamma \cdot \Gamma|0_i^+>$ in exact equivalence with Eq. (6.8) and (6.9) of [12]. This 2ν double Gamow-Teller operator creates a pair of protons coupled to total angular momentum zero and annihilates two neutrons of the normal parity space coupled to pseudo orbital angular momentum and pseudo spin equal to zero. For the rare earth and actinide nuclei this operator annihilates two neutrons in the pseudo shells $\eta_\nu = 5$ and $\eta_\nu = 6$ and creates two protons in the abnormal orbits $h_{11/2}$ and $i_{13/2}$, respectively.

Appendix C: The finite nucleon size and short range correlation effects

In the momentum representation it is not difficult to include additional effects. The finite nucleon size (FNS) effects are introduced through the dipole form factor [3, 14]

$$v_{FNS}(p) = v(p) \left\{ \frac{\Lambda^2}{\Lambda^2 + p^2} \right\}^4 \quad \Lambda = 850 \text{ MeV} \quad (37)$$

The two-nucleon short range correlations (SRC) mainly arise from the repulsion due to the ω -exchange in the nucleon-nucleon interaction [32]. Thus the correlated potential is constructed as

$$\begin{aligned}
v_{SRC}(p) &= \int \frac{d^3k}{(2\pi)^3} \Omega(\vec{p} - \vec{k}) v(p) \\
\Omega(\vec{p} - \vec{k}) &\equiv (2\pi)^3 \delta(\vec{p}) - \frac{2\pi^2}{q_c^2} \delta(q - q_c)
\end{aligned} \tag{38}$$

with $q_c = 3.93 fm^{-1}$ roughly the Compton wavelength of the ω meson. Within the approximation $q_c \ll \bar{E}$ we get

$$v_{SRC}(p) = v(p) - \Delta v(p) \quad \Delta v(p) \equiv \frac{2\pi}{qq_c} \ln \left| \frac{q + q_c}{q - q_c} \right| \tag{39}$$

When both the FNS and the SRC effects are considered the neutrino potential takes the form:

$$v_{FNS+SRC}(p) = v_{FNS} - \Delta v(p) + \Delta v'(p) \tag{40}$$

with

$$\Delta v'(p) \equiv \frac{\pi}{pq_c} \left\{ \sum_{n=1}^3 \frac{1}{n} [x_-^n - x_+^n] + \ln \frac{x_-^n}{x_+^n} \right\} \quad x_{\pm}^n \equiv \frac{\Lambda^2}{\Lambda^2 + (p \pm q_c)^2}. \tag{41}$$

It was shown in Ref. [31] that the FNS effects are negligible for the allowed transitions but increases rapidly when L increases, and that, if both FNS and SRC are considered simultaneously the net result may not be very different from that obtained without any SRC at all.

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Table Captions

Table 1. Theoretical estimates for the nuclear matrix elements and the half-life of the $\beta\beta$ -decay in the 0ν mode for several heavy deformed nuclei are given, under the assumption $\langle m_e \rangle = 1eV$, and compared with the available experimental data.

Table 1

Transition	$M_{0\nu}$	$G_{0\nu}[yr s^{-1}]$	$\overline{E}[MeV]$	$\tau_{theo}^{1/2}$	$\tau_{exp}^{1/2}$
$^{146}Nd \rightarrow ^{146}Sm$	1.16	1.65×10^{-17}	13.5	1.18×10^{28}	
$^{148}Nd \rightarrow ^{148}Sm$	1.57	1.57×10^{-14}	13.6	6.75×10^{24}	
$^{150}Nd \rightarrow ^{150}Sm$	1.57	1.01×10^{-13}	13.7	1.05×10^{24}	$> 2.1 \times 10^{21}$ [16]
$^{186}W \rightarrow ^{186}Os$	1.70	1.76×10^{-15}	15.3	5.13×10^{25}	$> 2.3 \times 10^{20}$ [33]
$^{192}Os \rightarrow ^{192}Pt$	0.72	1.58×10^{-15}	15.5	3.28×10^{26}	
$^{238}U \rightarrow ^{238}Pu$	1.76	8.21×10^{-14}	17.3	1.03×10^{24}	$> 2.0 \times 10^{21}$ [10]

Figure Captions

Figure 1. The $M_{0\nu}^{GT}(j_\nu)$ (white columns) and $-M_{0\nu}^F(j_\nu)$ (black columns) components of the Gamow-Teller and Fermi nuclear matrix elements for ^{238}U , as function of the angular momentum of the decaying neutrons.

Figure 2. The same of Fig. 1 for ^{150}Nd .

Figure 1:

Figure 2:

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